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A dynamic antivibration system

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Abstract

In many sensitive instruments and production machines, the achievable resolution is limited by the ever-present building vibrations. A new dynamic isolation system is described that effectively removes these vibrations and has clear advantages over the traditional soft passive isolation system.

Introduction

In industry and research the quest for tighter production tolerances and higher resolution places stringent requirements on the environment. Vibration control is becoming more and more important, and better solutions are needed for the isolation of equipment from the ever-present building vibrations.

It is convenient to divide the spectrum of building vibrations into the two regions above and below about 200 Hz. In the high frequency region internal vibration modes of a piece of equipment may be resonantly excited either by direct transmission through the support structure or via pressure waves in the surrounding air. In general these resonances do not pose a severe problem. They can normally be avoided by simple high frequency isolation techniques (rubber pads under the support structure, for example) and if necessary an acoustically damped enclosure. The low frequency vibrations are, however, much more serious. They produce a non-resonant distortion proportional to the acceleration involved. The most disturbing effects occur typically in the frequency range of 10-30 Hz where the majority of buildings have their largest vibration amplitudes (accelerations of 10^{-3} to 10^{-2} g in this frequency range are normal).

It is these low frequency vibrations that will be addressed in this paper. Although passive isolation systems have been developed for this frequency range, they have many shortcomings. Previous attempts[1] to build dynamic isolation systems have notsucceeded in making the jump from drawingboard to practice. A new dynamic isolation system has been constructed[2] which offers significant improvements over the available passive system.

Passive Isolation

The simplest method of vibration isolation consists of mounting the device to be isolated on a resilient support. Consider such a mount consisting of a mass M supported on a spring of force constant λ as illustrated in Fig. 1(a). Of interest is the vertical

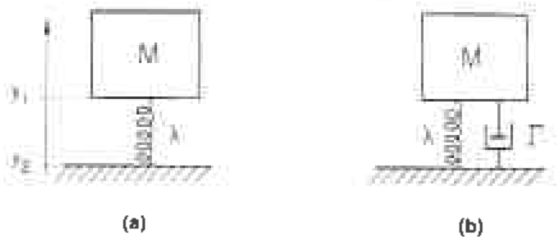

Figure 1(a) Simple isolation and (b) passive isolation with damping.

movement y_1 of the mass M in response to a movement y_2 of the supporting surface. If y_2 is a harmonic displacement with angular frequency ω and amplitude a_2, then the response y_1 is also harmonic with amplitude a_1:

$$y_1 = a_1 e^{i\omega t},$$

where it is straightforward to show that

$$\frac{a_1}{a_2} = \frac{1}{1 - \frac{M\omega^2}{\lambda}} \quad (1)$$

At high frequencies, the second term in the denominator is large and a_1 therefore is small, i.e., high-frequency isolation is good. However, at a frequency ω_o given by $M\omega_o^2/\lambda = 1$, the amplitude a_1 becomes infinite, and some damping must clearly be introduced in a practical device.

In the damped system of Fig. 1(b) there is a viscous force F acting with amplitude

$$F = \Gamma\lambda \frac{d}{dt}(y_1 - y_2),$$

and the response of the damped system is

$$\frac{a_1}{a_2} = \frac{1 + i\Gamma\omega}{\left(1 - \frac{\omega^2}{\omega_o^2}\right) + i\Gamma\omega} \tag{2}$$

The amplitude a_1 is now complex (phase shifted), but the amplitude at resonance is reduced to

$$\left|\frac{a_1}{a_2}\right|_{\omega = \omega_o} = \left(1 + \frac{1}{\Gamma^2\omega_o^2}\right) \tag{3}$$

In practice the choice of damping Γ is a compromise between low resonant amplitude (Eq. 3) and sufficient high-frequency isolation. Eq.(2) shows that for the high values of Γ needed for low resonant amplitude, the amplitude a_1/a_2 tends to unity, i.e., there is no attenuation of the vibration amplitude. A practical compromise is obtained with $\Gamma \simeq 1/(3\omega_o)$.

While more complex spring systems can offer better high-frequency isolation, the problem of the buildup of amplitude at resonance remains.

Dynamic Isolation

The dynamic antivibration system to be described here comprises units consisting of the combination of an accelerometer with an electromechanical transducer. Such a unit is illustrated in Fig. 2. The accelerometer consists of a mass m resting on a piezoelectric disc P, the whole being supported by a bracket B on the device of mass M to be isolated. The electromechanical transducer LS (e.g., a modified loudspeaker) allows a variable force to be applied to the underside of the bracket. The force and the measured component of acceleration are collinear. In parallel with the electromechanical transducer is a spring of force constant λ which supports the weight of the body to be isolated.

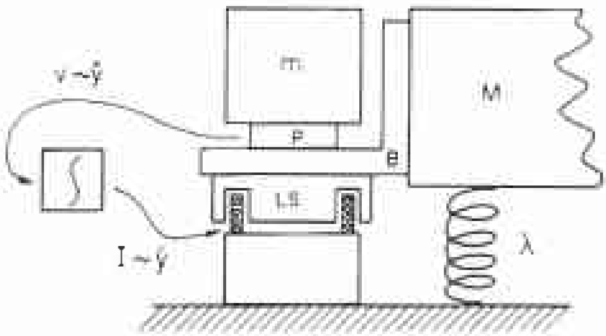

Figure 2. The principle of dynamic vibration isolation.

Since the mass m is supported only by the piezoelectric disc P, the voltage V developed across P is proportional to the absolute acceleration (d^2y_1/dt^2) of the mass m. This voltage is integrated to produce a current $I \sim \int V dt$, which produces a force $k_2 I$ in the electromechanical transducer, where k_2 is a constant. The equation describing the motion of the unit is therefore (assuming $M \gg m$):

$$M\frac{d^2y_1}{dt^2} = -\lambda(y_1 - y_2) + k_2 I.$$

where

$$I = \int V dt = m \int \frac{d^2 y_1}{dt^2} dt = m \frac{dy_1}{dt}$$

or

$$k_2 I = -k_1 m \frac{dy_1}{dt},$$

where k_1 is a constant determined by the gain of the integrator. Therefore

$$M \frac{d^2 y_1}{dt^2} = -\lambda(y_1 - y_2) - k_1 m \frac{dy_1}{dt},$$

from which it is straightforward to show that the amplitude a_1 of the response to a driving amplitude $y_2 = a_2 \exp(i\omega t)$ is

$$\frac{a_1}{a_2} = \frac{\lambda}{(\lambda - M\omega^2) + ik_1 m\omega}. \qquad (4)$$

The amplitude a_1 therefore decreases with increasing values of k_1 (gain).

By making k_1 large enough the resonant term in the denominator becomes negligible, except at very low and very high frequencies. The ratio a_1/a_2, or transmissibility, is plotted in curve (a) of Figure 3 and shows two frequency intervals. As ω tends to zero the transmissibility tends to unity. An intermediate regime follows in which the active feedback is dominant and the transmissibility tends to

$$\frac{a_1}{a_2} = \frac{\lambda}{ik_1 m\omega}$$

At the highest frequencies(not shown)

$$\frac{a_1}{a_2} = \frac{-\lambda}{M\omega^2}$$

corresponding to the passive isolation afforded by the spring mount.

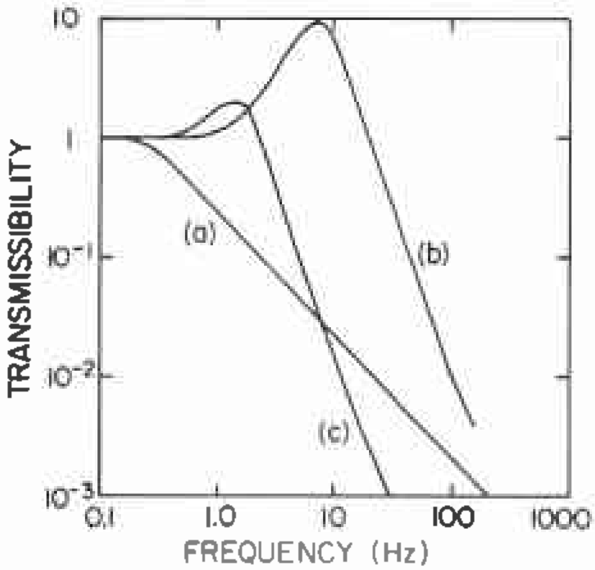

Figure 3. Transmissibility of dynamic isolation system curve (a), compared with passive isolation, curve (b) for a simple spring and curve (c) for a compound spring.

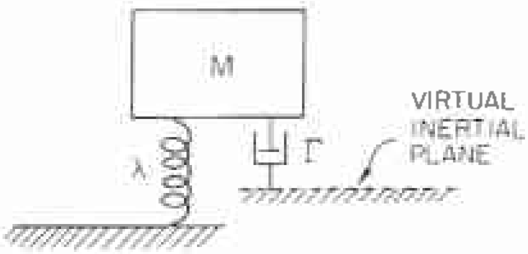

Figure 4. Illustration of damping with respect to a virtual inertial plane.

For comparison the transmissibility of the simple damped spring (equation 2) is plotted in curve (b) and of a good commercially available compound spring with $\omega_o/2\pi$ about 1.5 Hz (air columns) in curve (c). The comparison shows the complete lack of resonant behaviour of the dynamic system. Furthermore the isolation of the dynamic system is better at all frequencies than the simple damped spring and is inferior to the compound spring only at high frequencies where both systems isolate so well that the difference is unlikely to be of importance.

It is of interest to understand the physical reason underlying the lack of resonant behaviour in the dynamic isolation system. The damping term in equation (4) appears only in the denominator and may therefore be made arbitrarily large, unlike for the passible case where the damping term appears both in the numerator and in the denominator. By following the derivation of the equations, it will be seen that this distinction arises because for the passive case damping is proportional to the relative velocity between mass and floor, while for the dynamic case the damping is proportional to the absolute velocity. An absolute velocity may be thought of as a velocity relative to a virtual inertial plane. Thus, as illustrated in Fig. 4, high damping in the dynamic system is equivalent to strong coupling to an inertial reference frame and, therefore the stronger the coupling the better the isolation.

The Dynamics of the Supported Mass

Consider the effect of a force $F = F_o e^{i\omega t}$ applied directly to the supported mass M. It is easy to show that for the passive case, a displacement a_1 is produced where

$$a_1 = \frac{F_o}{M(\omega_o^2 - \omega^2 + i\omega_o^2\omega\Gamma)}, \tag{5}$$

At frequencies $\omega \gg \omega_o$ this tends to

$$a_1 = -F_o/M\omega^2$$

in other words the supported mass behaves like an unconstrained or free mass M. At very low frequencies a_1 tends to the constant value $Fo/M\omega_o^2$ ($= F_o/\lambda$) reflecting the fact that the mass is compliantly coupled to the ground. Since ω_o is low, a_1 is large, corresponding to high compliance. The response of a passively mounted mass to an applied force varies therefore from that of a free mass at high frequencies to that of a mass-less highly compliant spring at very low frequencies. Around the resonant frequency ω_o the response is greater than for a free mass M and phase shifted. A simple example of a body showing similar dynamic behaviour to that described above is a body with density just less than unity floating in water. The term "floating" is commonly used to describe the behaviour of a passively supported body.

The dynamic response of the actively isolated system has the same form as equation 5, namely

$$a_1 = \frac{F}{M\left[\omega_o^2 - \omega^2 + i\omega\,\frac{k_1 m}{M}\right]} \tag{6}$$

However the behaviour described by equation 6 is very different for two reasons:

1) ω_o^2 is typically 400 times larger, (see below), and
2) the damping term is up to two orders of magnitude larger than for the best passive system.

The dynamic response of the actively isolated mass M is therefore equivalent to that of a mass M sitting on a very stiff and highly overdamped spring. Because of the large spring constant the mass appears to be rigidly attached to the floor, and because of high damping the dynamic response to an applied force F is much smaller than that of a free mass M.

It is tempting to describe the dynamic response of the actively isolated mass as being that of a very much larger free mass, in other words of an effective mass M' many times the actual mass M. Equation 6 shows, however, that this is strictly incorrect. Since the damping term is dominant, the dynamic response must be described as that of a body moving in a highly viscous medium, and not that of a large free mass.

Detailed Construction

In designing a practical device based on the above principle two points must be borne in mind. First, the device discussed above isolates in one direction (and at one location) only. A minimum of six such devices must be attached to the body to be isolated since the body has six degrees of freedom (3 translational, 3 rotational).

Second, the gain of the feedback loop cannot, in practice, be increased indefinitely. At some point the gain at high frequencies, where mechanical resonances cause phase shifts, will be high enough to cause oscillation. Rigid construction is required to place these mechanical resonances at sufficiently high frequencies. Note that this applies mainly to the unit comprising the accelerometer and electromechanical transducers although resonances in the supported body can be troublesome if they are not well damped and occur at frequencies where the feedback loop still has significant gain. This latter suggests an upper frequency at which the gain in the feedback loop should fall to unity of the order of 200-300 Hz.

The lower frequency limit may be set around 1/2 - 1 Hz for most purposes since buildings have very little vibration amplitude below about 5 Hz. There are more practical considerations - the chosen accelerometer becomes noisy at very low frequencies and high gain electronics with very low frequency response may have annoyingly long settling times.

Having chosen upper and lower cut-off frequencies of about 300 and 1/2 Hz respectively, one must determine the spring constant of the passive support structure. This can be determined by considering the open loop gain G of the feedback loop which has the form

$$G = \frac{-K i \omega}{1 - \frac{\omega^2}{\omega_0^2} + i\Gamma\omega}$$

where K is a constant and Γ is a small damping term associated with the passive support structure. It is seen that the gain is maximum at the mechanical resonance ω_o of the system without feedback, and the feedback is exactly out of phase. At lower frequencies the gain falls off proportional to ω and the feedback signal leads by 90^o. At higher frequencies the gain falls off proportional to $1/\omega$ and the feedback signal lags by 90^o.

Taking the frequency limits as those frequencies at which G = +/- i we find approximately ω_{lower} = G/K and ω_{upper} = $K\omega_o^2/G$ from which it is seen the mechanical resonant frequency ω_o of the system without feedback should be chosen to lie near the geometric mean of the extreme operating frequencies of the feedback loop, i.e. in the range $\omega_o/2\pi$ of 10-20 Hz.

In principle the dynamic isolation elements can be attached to any rigidly constructed and resiliently mounted body. The design below applies to a general purpose table that, in its prototype form is capable of supporting loads up to about 300 kg, but it can be straightforwardly scaled to one that will support larger loads.

Referring to Fig. 5, the table top is supported near its corners on four supports, each consisting of a resilient rubber block R and a base S. The thickness and area of the rubber blocks are chosen so that the vertical resonant frequency of the table top plus supported mass lies in the range 10-30 Hz.

Antivibration elements are attached at various points to the table top as indicated by the arrows. A minimum of six are required but more may be used, for example, to increase the correction forces available. A convenient arrangement involving eight elements is illustrated.

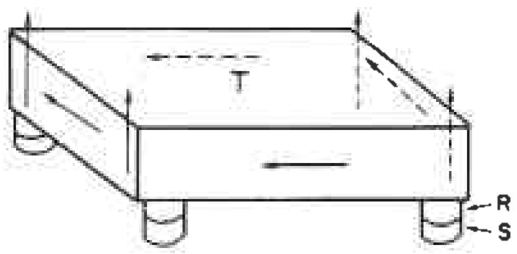

Figure 5 Resiliently mounted table top with eight antivibration elements indicated by the arrows.

The design of each element is illustrated in Fig.6. A bracket B attached to the table top supports an electromechanical transducer LS on its lower surface and an accelerometer A on its upper surface. The transducer LS supplies a force proportional to the applied current. For this purpose a loudspeaker is used with a block C glued to the center of the cone in intimate contact with the voice coil. The force provided by the transducer and the component of acceleration measured by A both lie along the common axis of A and LS. Since the force must act between the floor and the bracket B, the end of the transducer remote from the bracket B must be coupled to the floor in such a manner that the coupling

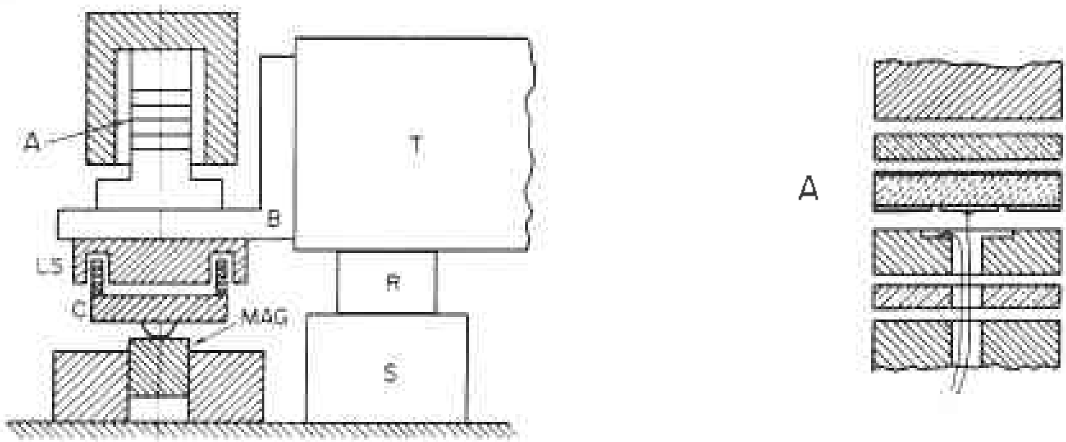

Figure 6 (a) Detailed construction of antivibration element and (b) exploded view of thermally compensated accelerometer.

is rigid in the direction of the common axis of A and LS and compliant perpendicular to this direction. The transverse compliance is simply achieved by attaching to the block C a steel ball, which rests on and is firmly held by the magnet (MAG). During a transverse displacement, the block C plus ball rocks on the magnet, causing a rotation of the loudspeaker voice coil. During an excessive transverse displacement, the ball may slide on the magnet, thus preventing damage to the loudspeaker.

The magnet itself is held in a friction coupling which allows it to slide along the common axis of A and LS whenever a certain force is exceeded. Clearly this frictional force must be less than the magnetic coupling force between ball and magnet, but must be greater than the maximum force generated by the transducer LS. The above combination of ball, magnet and friction coupling forms a completely self-aligning coupling between the floor and the transducer LS.

The vibration element depicted in Fig. 6 acts in the vertical direction. For the horizontally acting elements of Fig. 5, the bracket B supporting the element is rotated through 90° and the friction coupling holding the magnet is appropriately altered. The following qualities are required of the accelerometer:

(a) The lowest mechanical resonance must exceed 10 kHz so that adequate gain may be applied in the feedback loop. To achieve this, the PZT must be used in the compressional mode; bending modes (bimorph devices) are too soft.

(b) The PZT element is a pure capacitance C and requires a resistance R in parallel to define the dc conditions. The lowest frequency, 1/RC, to which the element will respond must be chosen lower than the lowest frequency component of the vibrations to be compensated. The resistance R causes a mean square noise voltage

$$\overline{\Delta V^2} = 4\,kTR\Delta f, \text{ where } \Delta f \propto 1/RC, \text{ so that } \overline{\Delta V^2} \propto \frac{4kT}{C}$$

For typical values of C ~ 1 nF, the noise voltages are on the order of about 4 μV. The mass m must be chosen large enough so that the smallest accelerations to be measured will give signals larger than this limiting noise value. For the design considered here this would require m to be about 5 kg which is too high for convenience. A compromise was reached with m = 1 kg and some degradation of performance below 5 Hz.

(c) PZT material is also pyroelectric, typically producing signals of 10 V per degree of temperature change. A change of 1 μK would produce a signal larger than the thermal noise voltage discussed above; it is therefore necessary to thermally isolate the PZT and to use some thermal compensation.

The detailed accelerometer construction is shown in Fig. 6(b). The silver-coated PZT disc is axially polarized and attached on one surface to a glass disc which supports a boss for attaching the mass m. The other surface of the PZT has an annulus of the silver coating removed. A steel thrust piece is attached to the outer ring of the PZT. Electrical contacts are made to the thrust piece and the center of the PZT. The whole construction sits on another glass disc (for thermal isolation), which in turn sits on the accelerometer base.

Notice that a temperature change will produce equal and opposite charges on the two faces of the PZT disc but will produce no voltage difference between the outer ring and center of the lower surface of the PZT. On the other hand, an acceleration will cause a change of force across the outer ring of the PZT only and will, therefore, produce a voltage difference.

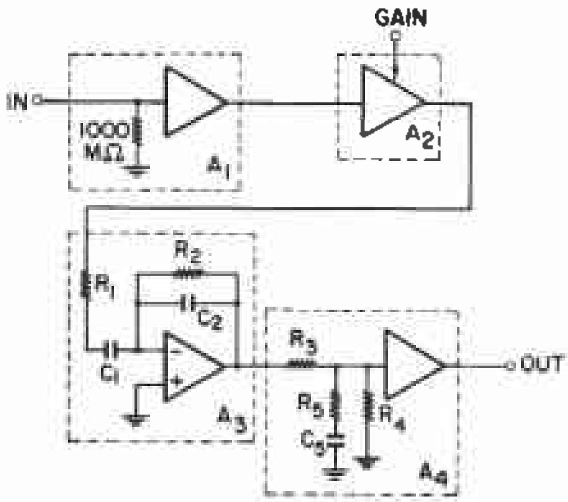

Figure 7. Block diagram of electronic circuit for dynamic vibration isolation.

The accelerometer signal is procesed by the circuit shown in Fig. 7 before being fed back to the electromechanical transducer LS as the correction signal. The basic processing consists of integration, with some low- and high-frequency attenuation. A preamplifier A_1 with 1000 MΩ input impedance is followed by a variable gain stage A_2 employing a transconductance operational amplifier. The signal is then fed to the integrator A_3. DC blockig by the capacitor C_1 passes only signals with frequency higher than a predetermined low-frequency limit (typically 1 Hz) and thus, together with resistor R_2, maintains dc stability. The time constant C_2R_2 should be roughly one second or greater. The output of the integrator is finally amplified by the power stage A_4 before passing to the transducer. The divider chain comprising R_3, R_4, C_5, R_5 gives high-frequency attenuation above a frequency of around 200 Hz and allows higher gain to be used in the feedback loop before the onset of high-frequency oscillation. Note that this high frequency compensation is a compromise. The stability of the loop is decreased in the range 200-600 Hz and allows structural resonances in the supported body to lead to resonances in the transmissibility in this range. The low frequency gain is, however, thereby increased.

The power stage A_4, illustrated in greater detail in Fig. 8, is a class D switch-mode amplifier giving a very high power-converion efficiency. The waveform V_1, V_T, and V_O are shown in Fig. 9. The diodes D_1 introduce a nonlinearity at large input voltages, which prevents the stage from being driven into saturation.

The stage can deliver up to 30 W but under normal conditions (in a laboratory with a maximum acceleration of around 10 cm/sec^2 at 20 Hz) the power required is under 1 Wper isolation element. Full power is normally only required for isolating against forces supplied directly to the supported mass M (e.g., hand contact). Because of the large difference between mean power and maximum power it is convenient to use rechargeable batteries as the power source with a trickle charger providing sufficient power to cover the mean consumption.

As mentioned above, the PZT accelerometer is noisy at very low frequencies. While this in itself cannot cause instability in the loop, the electronics will see this as an apparent acceleration and try to reduce it. As a result the feedback loop has to work hard and an opposite acceleration will be produced in the table top. To avoid this some attenuation has been introduced into the loop (not shown in Figure 7) below about 5 Hz, but at a price - the noise movements are no longer troublesome but the extra phase lead introduced by the attenuation makes the loop only marginally stable and leads to a weak resonance in the transmissibility around 1 Hz. This whole problem would be avoided and low frequency isolation improved by use of noise free accelerometers.

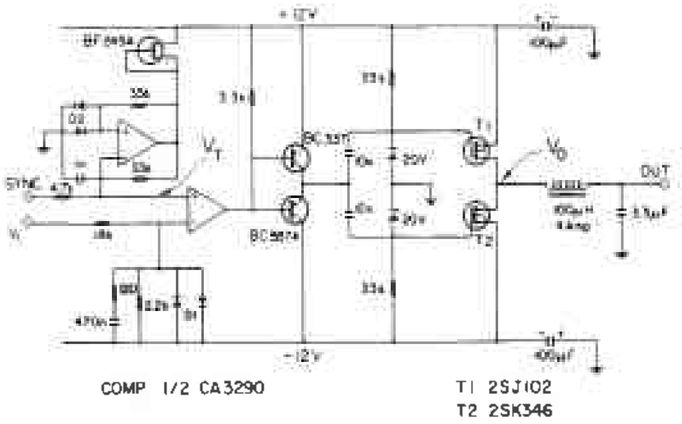

Figure 8 Class D output stage

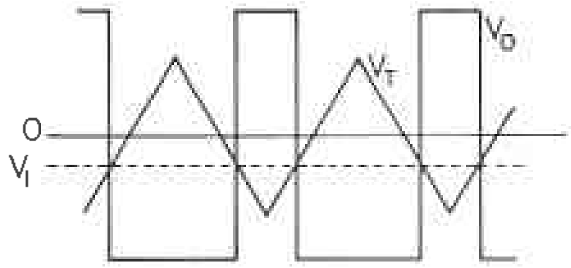

Figure 9. Waveforms associated with the output stage of Fig. 8

Isolation Properties

The transmissibility of the system is plotted in Figure 10 and compared with the ideal curve of Figure 3. The horizontal isolation is the same as the vertical. The discrepancy between the two curves at low frequencies is entirely due to the extra attenuation introduced to avoid the accelerometer noise. This fact cannot be too strongly emphasiszed - it is not an intrinsic property of the active isolation system. Subsequent experiments

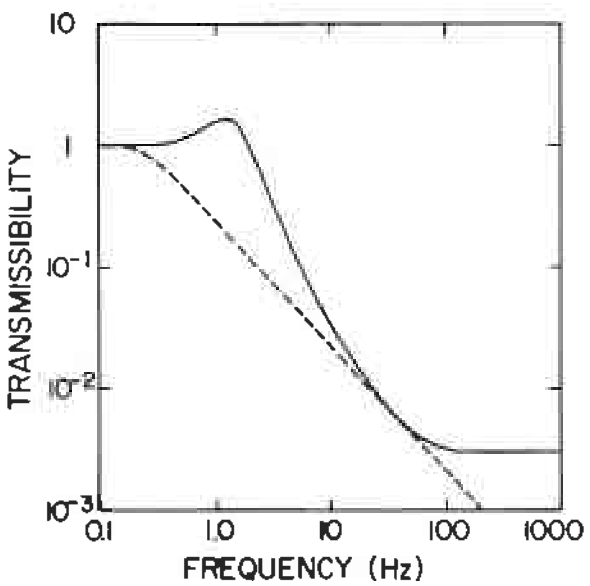

Figure 10. Measured transmissibility (solid line) compared with that for the ideal system using noise-less accelerometers.

using different detectors have shown that the ideal isolation curve can indeed be achieved.

Applications

The table described here, using dynamic vibration isolation, is essentially rigid at frequencies greater than about 1 Hz. Thus, when used as a support for a high-resolution optical or electron microscope, forces introduced by the operator's hands or pumping lines are absorbed without producing movement in the table top. Similar arguments apply when it is used as a support for a spectrometer; a Fabry-Perot interferometer has been operated on a predecessor of the present table for several years with quite unprecedented resolution. We have high expectations that the table could be effectively used as a support for step-and-repeat cameras, which are presently running into problems as resolution is pushed to the limits. The dynamic rigidity offered by this table should closely approximate the ideal rigid base on which instruments are designed to operate.

The following paper by J. Turechek will discuss in detail different areas of application of the system.

Acknowledgements

It is a pleasure to thank M. Tgetgel and E. Meier for their advice and help in constructing the system.

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2. Sandercock, J.R., Tgetgel, M., Meier, E., "A Dynamic Antivibration Support", RCA Review, Vol. 46, pp. 70-80. 1985.